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# Vorwort

Das Tätigkeitsfeld des Fraunhofer-Instituts für Techno- und Wirtschaftsmathematik ITWM umfasst anwendungsnahe Grundlagenforschung, angewandte Forschung sowie Beratung und kundenspezifische Lösungen auf allen Gebieten, die für Techno- und Wirtschaftsmathematik bedeutsam sind.

In der Reihe »Berichte des Fraunhofer ITWM« soll die Arbeit des Instituts kontinuierlich einer interessierten Öffentlichkeit in Industrie, Wirtschaft und Wissenschaft vorgestellt werden. Durch die enge Verzahnung mit dem Fachbereich Mathematik der Universität Kaiserslautern sowie durch zahlreiche Kooperationen mit internationalen Institutionen und Hochschulen in den Bereichen Ausbildung und Forschung ist ein großes Potenzial für Forschungsberichte vorhanden. In die Berichtreihe sollen sowohl hervorragende Diplom- und Projektarbeiten und Dissertationen als auch Forschungsberichte der Institutsmitarbeiter und Institutsgäste zu aktuellen Fragen der Techno- und Wirtschaftsmathematik aufgenommen werden.

Darüber hinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation des Transfers aktueller Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte – und umgekehrt, denn Probleme der Praxis generieren neue interessante mathematische Fragestellungen.

A handwritten signature in black ink, appearing to read 'Dieter Prätzels-Wolters'.

Prof. Dr. Dieter Prätzels-Wolters  
Institutsleiter

Kaiserslautern, im Juni 2001



# Planning for Home Health Care Services

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## Abstract

In this paper we are looking at routing and scheduling problems arising in the context of home health care services. Many small companies are working in this sector in Germany and planning is still done manually, resulting in long planning times and relatively inflexible solutions.

First, we consider the home health care problem (HHCP) which seeks for a weekly optimal plan. However, in practice a master schedule is generated which is modified to operational changes. We take this approach into account by looking at the master schedule problem (MSP) and at the operational planning problem (OPP).

The problems are solved using different metaheuristics combined with methods from constraint programming. This allows a very flexible treatment of realistic constraints.

Computational results are presented using real world data.

**Keywords:** Home Health Care, Route Planning, Metaheuristics, Constraint Programming

## 1 Introduction

Home Health Care (HHC) services are becoming increasingly important in Europe's aging societies. Elderly people have varying degrees of need for assistance and medical treatment. It is advantageous to allow them to live in their own homes as long as possible, since a long-term stay in a nursing home can be much more costly for the social insurance system than a treatment at home providing assistance to the required level. Therefore, HHC services are a cost-effective and flexible instrument in the social system.

In Germany, organizations providing HHC services are generally either larger charities with countrywide operations or small private companies offering services only in a city or a rural area. While the former have a hierarchical organizational structure and a large number of employees, the latter typically only have some ten to twenty nurses under contract. The relationship to the patients ("customers") is often long-term and can last for several years. Therefore acquiring and keeping satisfied customers is crucial for HHC service providers and intensive competition among them is observed.

Due to this competition, it is very important for HHC services to optimize their operational costs. Two main operational parameters contribute to these costs: overtime work and length of tours to be travelled to the patients homes. Therefore operational planning in HHC services tries to minimize these cost factors while providing a good service level to the customers, e.g., seeing them at their desired time of the day by a nurse they know well and have confidence in.

Operational planning at HHC services is nowadays mostly done manually—often by an experienced senior nurse. The planning problem is quite complex. Rosters for nurses and tours have to be

planned with regard to providing a good level of service and minimizing costs. To make this difficult task solvable, HHC services typically work with a mid-term plan (“master schedule”) that is the basis for the day-to-day operational planning. Daily adaptations are made due to non-availability of personnel or changes in the jobs to be performed, e.g., if a patient has been taken to the hospital or has returned to his home. The master schedule is less frequently updated, mainly to take into account new or cancelled contracts with patients.

The complexity of HHC operations planning makes it an interesting problem for planning support by Operations Research techniques. Interestingly, to our knowledge only a few papers deal in particular with HHC planning. The problem has first been described by Cheng and Rich in 1998 [4]. More recent research was performed by Bertels and Fahle [2] and by Eveborn, Flisberg and Rönnqvist [6].

Cheng and Rich consider a home health care model with full-time and part-time nurses. For each nurse, a lunch is scheduled with a fixed duration and a hard time window. The problem is tackled with a two-phase construction heuristic: in the first phase, the tours of the nurses are parallelly built according to a greedy heuristic. In the second phase, the schedules are tightened. Randomly generated and real-world data is used to test the proposed algorithm.

A more recent article by Bertels and Fahle combines constraint programming (CP) with meta-heuristics to solve the HHCP. In their model, hard and soft time windows determine the time for the performance of a job and the availability of a nurse. The model is solved with a two stage approach: in the first stage, they partition the jobs into sets that have to be performed by one nurse. Afterwards, for each job set, an order is generated. The algorithm was applied to artificial test instances of 20 to 50 nurses with 200 to 600 jobs to be scheduled.

The last paper by Eveborn, Flisberg, and Rönnqvist models the HHCP as a set partitioning problem consisting of visits on the one side and staff members on the other side. The goal is to match visits to staff members such that the constraints are satisfied. The problem is solved with a repeated matching algorithm. Only a few test results are presented. Instances with up to 20 staff members and 123 visits were solved within 140 seconds.

Our paper considers HHC operations planning from several perspectives. Firstly, we introduce the home health care problem (HHCP) as a full-sized model for HHC planning and present a hybrid solution method for it in Section 2. Our second model takes the current planning process in practice into account and considers the construction of good master schedules (the master schedule problem, MSP). This model abandons the requirement of providing rosters for the nurses. Therefore we can adapt and simplify the heuristic for the HHCP to solve it (Section 3). As the MSP is intended for mid-term planning, the operational planning problem (OPP) considers the requirement to incorporate last minute changes into an existing plan. Our essential aim is to limit perturbations to this plans, thus we formulate stability of the plan as an explicit objective in the OPP. To solve the OPP, we provide a hybrid solution algorithm (Section 4).

In Section 5, we evaluate the models and methods with real-world data from Germany and the Netherlands. The results show that with suitable models and algorithms it is possible to solve practical instances of HHC operations planning in reasonable time. In our conclusion (Section 6), we summarize our findings and give directions for further research.

## 2 The Home Health Care Problem

The task of the home health care problem (HHCP) is to create a service plan with nurses and patients such that the patients are served with the provided nurses. The HHCP is *NP*-hard. For this problem, we start from scratch, i.e., no historic plan or previous assignment is given.

Although the HHCP is no online problem, computation time is limited. Typically a senior nurse wants to solve the HHCP Friday morning for the next week. Consequently, the allowed computation time is a few hours such that the nurse may respond to possible infeasibilities. The senior nurse prefers a method that provides her with a feasible solution as soon as possible. Afterwards, additional computation time is usually available to improve the solution.

To meet these demands, we propose a two stage approach. In the first stage, a constraint programming heuristic provides a good feasible solution within seconds or a few minutes. If additional

Table 1: Parameters for a job  $j$  in the HHCP

$[hbs_j, hbe_j]$	Hard time window
$pt_j$	Fixed service duration
$f_j$	Frequency
$R_j$	Possible shift combinations,
$req_j$	Qualification required by a nurse
$d_{jk}$	Distance from job $j$ to job $k$

computation time is available, a hybrid constraint programming-adaptive large neighborhood search seeks to improve this initial solution.

Constraint programming (CP) is a mathematical paradigm that infers information about feasible solutions from a problem’s constraints. It allows the forward detection of infeasibilities in a branch and bound search.

For the second stage, we apply an adaptive large neighborhood search (ALNS) [10]. In contrast to traditional meta-heuristics like tabu search or simulated annealing, the ALNS traverses the search space by significantly altering the current solution. This behavior allows to overcome local optima. Throughout the ALNS, a CP layer maintains the feasibility of the newly generated solutions.

In the course of this section we describe the model and the predetermined constraints, before we sketch the solution approach in more detail.

## 2.1 Model

Let a set of shifts  $\mathcal{S} = \{1, \dots, S\}$ , the planning horizon, be given. Each shift has a hard time window  $[0, H]$ , where the term “hard” means that a bound or a constraint cannot be violated—not even by paying penalty costs. In contrast to the shifts we speak of a nurse-shift whenever a nurse works a shift. If for example a nurse works Mondays and Fridays, she creates two nurse-shifts. The planning horizon of a home health care service is usually a week.

The patients are represented by a set of jobs  $\mathcal{J} = \{1, \dots, J\}$ . A job  $j$  represents a duty at a patient’s home which is characterized by a hard time window  $[hbs_j, hbe_j] \subseteq [0, H]$  in which it has to be performed. A nurse has to stay with a patient for the service duration  $pt_j$ . The service duration and the time window of a job are identical for all shifts. Furthermore, each job has a frequency  $f_j$ , which states in how many shifts the job must be performed during the planning horizon. The shifts in which it has to be planned are given by a set of possible shift combinations  $R_j = \{R_{j,1}, \dots, R_{j,K_j}\}$ . The sets  $R_{j,l}$  are subsets of shifts,  $R_{j,l} \subseteq \mathcal{S}$ , with the characteristic that  $|R_{j,l}| = f_j$  for all  $l \in \{1, \dots, K_j\}$ .

Each job belongs to exactly one patient, but one patient may create several jobs, e.g., washing in the morning, application of medication, or cleaning of the house. A job  $j$  has to appear  $f_j$ -many times in the schedule. Each of these occurrences is called a task. Therefore one patient may create several jobs which create several tasks.

The model contains a single depot. Each route of a nurse has to start and end at the depot within the time window of the shift. For this purpose, the depot is represented by a special job with index 0 and time window  $[0, H]$ . The set of jobs including the depot is denoted by  $\mathcal{J}_0 = \mathcal{J} \cup \{0\}$ .

Finally, a distance matrix identifies the location of the patients’ homes. The distance matrix

$$D = (d_{ij})_{i,j \in \mathcal{J}_0}$$

states the distance between two jobs  $i$  and  $j$  in time units. It is assumed that the distance matrix is symmetric and satisfies the triangle inequality, i.e.,

$$d_{ij} \leq d_{ik} + d_{kj} \quad \forall i, k, j \in \mathcal{J}_0.$$

The second main element of the HHCP are the nurses. Let  $\mathcal{N} = \{1, \dots, N\}$  be the set of nurses. A nurse is a staff member qualified to perform tasks. Each nurse  $n \in \mathcal{N}$  has an a priori

Table 2: Parameters for a nurse  $n$  in the HHCP

$a_n^s$	1, if available in shift $s$ ; 0, otherwise
$wt_n$	Working time of the nurse covered by contract
$Q_n$	Qualifications provided by the nurse
$c_n$	Cost for each overtime unit

availability  $a_n^s \in \mathbb{B}$  for all  $s \in \mathcal{S}$  with  $a_n^s = 1$ , if nurse  $n$  is available for shift  $s$ , 0 otherwise. If a nurse is unavailable in this sense, she cannot be assigned to any task in this shift—not even by paying penalty costs. This unavailability occurs for example if a nurse is on holidays or sick.

Furthermore, a nurse has a designated working time  $wt_n$  for the whole planning horizon. For any working time beyond  $wt_n$ , a penalty of  $c_n \in \mathbb{N}_0$  is charged per additional time unit.

Due to the restrictions or a lack of nurses, it is possible that not all jobs can be assigned to a nurse. The introduction of a dummy nurse allows us to obtain a feasible solution in this case.

A dummy nurse is a special nurse which is not restricted to any constraints, i.e., multiple jobs can be assigned to her at any time, she is available at all times, etc. Only one dummy nurse is present in the model. For every task assigned to her, a penalty is added to the objective function.

Another aspect of our model is that not all jobs can be performed by just any nurse, e.g., a special medical treatment may require a specific training of the nurse. Therefore we introduce qualifications.

Let  $\mathcal{Q} = \{q_1, \dots, q_Q\}$  be the set of possible qualifications. Then, on the one hand, each nurse  $n$  provides a set  $\mathcal{Q}_n \subseteq \mathcal{Q}$  of qualifications, while on the other hand, each job  $j$  requires a qualification  $req_j \in \mathcal{Q}$ .

Consequently, an assignment of nurse  $n$  to job  $j$  is only feasible if nurse  $n$  provides qualification  $req_j$ , i.e.,  $req_j \in \mathcal{Q}_n$ . Typically the qualification levels are hierarchical, e.g., a nurse is capable of providing all qualifications that an assistant nurse can provide, while the opposite is typically not true. However, these hierarchies can occur in several dimensions: Assume a home health care service employs a nursing staff and a cleaning staff, then a member of the cleaning staff cannot perform any medical treatments, while a nurse is not supposed to clean.

The constraints for the HHCP are divided into two groups: satisfaction constraints and vehicle routing constraints. As satisfaction constraints, we have constraints satisfying the demand of the jobs. For each job, exactly one of its shift combinations is chosen, and a task representing the job is scheduled in each of those jobs. Exactly one nurse is assigned to each of these tasks. Finally, a job must be performed as often as indicated by its frequency.

A second group of constraints ensures feasibility for the nurses. Firstly, a nurse can only be assigned to a shift if she is available for this shift, and secondly, a nurse can only be assigned to a task if she possesses the required qualification.

Finally, we have the vehicle routing constraints. As stated before, each route has to start at the depot and end at the depot. Furthermore if a nurse arrives at a patient, she also has to leave this patient. All tasks are performed within their time windows, and subtours are forbidden for the routes.

We denote a tour as a subtour if it does not start at the depot, or if it does not end at the depot, or if it visits one or more tasks more than once.

## 2.2 Objective Function

Various objectives come to mind for the HHCP. On the one hand there are traditional vehicle routing objectives like minimizing the travel distance or the travel costs, while on the other hand there are practical concerns like the number of unscheduled jobs or the overtime costs. However, discussions with practitioners showed that customer satisfaction becomes more and more important. This customer satisfaction results from various factors like whether the nurses are always on time or whether the staff is friendly. Moreover, it is very important that only a few nurses visit one



patient such that the patient develops a personal relationship to the nurse. Therefore we introduce a patient-nurse loyalty.

The patient-nurse loyalty is an indicator of the continuity in a job's execution during the planning horizon. The patient-nurse loyalty  $l_j$  is defined as the number of different nurses performing the job during the planning horizon minus 1. A patient-nurse loyalty of  $l_j = 0$ , i.e., the patient is always treated by the same nurse, is the best value that can be achieved. As this value increases, the patient-nurse loyalty worsens.

In addition to the patient-nurse loyalty, we evaluate a created schedule by the number of unscheduled tasks, the overtime costs, and the traveling distance. Those four objectives are combined with a weighted sum, and we receive the following objective function that is to be minimized:

$$\min \quad \alpha_1 \cdot DNC \quad (1)$$

$$+ \alpha_2 \cdot \sum_{j=1}^J l_j \quad (2)$$

$$+ \alpha_3 \cdot \sum_{n=1}^N wc_n \quad (3)$$

$$+ \alpha_4 \cdot \sum_{n=1}^N \sum_{s=1}^S dt_n^s \quad (4)$$

In term (1),  $DNC$  represents the number of unscheduled tasks, while the next term sums up the patient-nurse loyalty penalty. Term (3) calculates the overtime with

$$wc_n = \max \left\{ 0, \sum_{s \in S} (end_n^s - start_n^s) - wt_n \right\} \cdot c_n.$$

The last term represents the distance traveled by all nurses throughout all shifts with  $dt_n^s$  being the travel distance of nurse  $n$  in shift  $s$ .

### 2.3 Two-Stage Solution Approach

Practitioners demand on the one hand that a feasible solution for the HHCP is computed quickly, while on the other one the solution obviously has to be applicable. These demands conflict with the computational complexity of the problem. To meet the practitioners' demands, we propose a two-stage solution approach.

In the first stage, a constraint programming heuristic guarantees the quick calculation of a feasible, applicable solution. Afterwards, an adaptive large neighborhood search (ALNS) seeks to improve the initial solution if further computation time is available. In the following, we first describe the heuristic before we turn to the ALNS.

Before we start with the heuristic, we reduce the complexity of the problem by assigning to each job its first shift combination. This assignment of shift combinations can only be revoked in the ALNS. Then we sort the tasks in each shift according to the earliest start times. The earliest start time  $est_j$  for a job  $j$  is given as

$$est_j = \max \{d_{0,j}, hbs_j\}.$$

Now we proceed with the heuristic shown in Figure 1.

At the beginning, we select the next shift  $s$ . If the shift still contains unscheduled tasks, we select the next task. If all tasks are scheduled in the shift  $s$ , we proceed with the next shift, until no more shifts with unscheduled tasks exist, i.e., we found a feasible schedule, and the heuristic ends. Otherwise, if we still have a task  $j$  to schedule, we select the next possible nurse for it. If no more nurses are available for this task, the task is assigned to the dummy nurse, i.e., it remains unscheduled. If a nurse  $n$  is available for task  $j$ , we create three branches in the search tree: First, we try to assign task  $j$  in shift  $s$  to nurse  $n$  ( $\text{Insert}(s, j, n)$ ). Then we branch that task  $j$

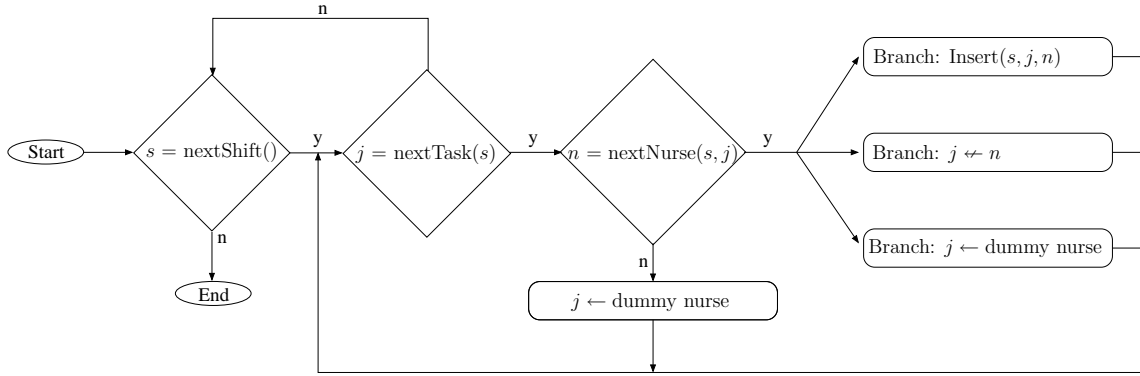


Figure 1: CP heuristic for the initial solution of the HHCP

cannot be performed by nurse  $n$  in shift  $s$  ( $j \leftarrow n$ ). Finally, we assign the task the dummy nurse ( $j \leftarrow \text{dummy nurse}$ ) if everything else failed.

For the performance of the heuristic and the quality of the solution, the selection of the next job and the next nurse is crucial.

For the selection of the next task, a simple idea has proven to be successful (see [1, Section 6.1]): assign the tasks according to their earliest possible start time. For this reason, we have sorted the tasks before we have started the heuristic. The assignment of the tasks with increasing earliest start times seeks to maintain a small number of unscheduled tasks.

The selection of the best nurse for the current task is more difficult. In order to maintain a good objective function value, we aim for small overtime costs and a small traveling distance. To achieve the former, we only allow nurses to be assigned to a task if they have not worked more than their individual average working time (total working time of the nurse divided by the total number of available shifts of the nurse) in the shift yet. The latter is achieved by assigning the nurse that is currently closest.

The heuristics for the selection of the next task and the selection of the best nurse lead to a tight initial schedule, which seeks to maximize the number of performed tasks, and hence minimizes the number of unscheduled jobs. The patient-nurse loyalty is not regarded in the initial solution, since it expresses complex relationships between cross-shift tasks. It is considered in the ALNS.

After the initial solution is found, an adaptive large neighborhood search (ALNS) is started. Within one of its iterations, called a move, a certain number of scheduled tasks is removed from the solution. For this purpose we maintain a set of several different removal operations. Examples for such operations are a random removal or a worst removal. While the random removal operation removes tasks randomly, the worst removal operation seeks to remove the tasks that influence the objective function most negatively.

The resulting infeasible solution is repaired with an insertion operation. The insertion operation re-inserts the previously removed tasks into the currently best position. Again, a set of different insertion operations is available.

During the course of the ALNS, we maintain a score for each removal and insertion operation that keeps track of the operations' success in the past. The success is measured according to whether the combination of the operations led to a new globally best solution, or it led to an improvement of the last solution, or it was accepted to diversify the search. The higher the score of an operation is the more likely it is that the operation is chosen for the upcoming move. This procedure leads to a preference of previously successful operations.

The ALNS (Algorithm 1) is called with two parameters: an initial solution  $s$  (in our case provided by the CP heuristic) and the number of tasks  $q$  that have to be removed from a solution. In the first step, the initial solution is stored as the globally best one. Then we iterate until a stopping criterion is met. This stopping criterion consists of a time limit and a move limit. If either one is reached, the search terminates.

---

**Algorithm 1** Pseudo-code for the Adaptive Large Neighborhood Search Algorithm [10]

---

```
1: function ALNS(solution  $s$ ,  $q \in \mathbb{N}$ )
2:   Solution  $s_{best} = s$ 
3:    $m = 0$  /* move counter */
4:   repeat
5:      $m = m + 1$ 
6:      $s' = s$ 
7:     Choose removal operation  $r$  according to the probabilities
8:     Choose insertion operation  $i$  according to the probabilities
9:     Remove  $q$  requests from  $s'$  with operation  $r$ 
10:    Reinsert removed requests into  $s'$  with operation  $i$ 
11:    if  $f(s') < f(s_{best})$  then
12:       $s_{best} = s'$ 
13:    if  $\text{accept}(s', s)$  then
14:       $s = s'$ 
15:    if  $(m \bmod \sigma) == 0$  then
16:      Update probabilities for the operations
17:  until (stopping-criterion is met)
18:  return  $s_{best}$ 
```

---

The actual move is described in lines 5 to 16 of Algorithm 1. After we increment the move counter, we store the current solution. Now we choose the removal and the insertion operation for the current move. In the next step we destroy the feasibility of the current solution by removing  $q$  requests according to the removal operation. Afterwards, a new feasible solution is created by the insertion operation. If the new solution has a new globally best objective function value (line 11), we store it as the best solution. Line 13 describes the function to evaluate the acceptance of  $s'$ , i.e., whether we use it in the next move as the current solution or not. For this purpose, we apply the acceptance procedure of simulated annealing heuristics [9], i.e., a solution  $s'$  is accepted as the new solution  $s$ , if it has an improved objective function value, or with a certain probability that decreases during search. Finally, we update the probabilities of the removal and insertion operations in every  $\sigma$ -th move, where the parameter  $\sigma$  is given a priori.

For more details on the algorithm refer to [11].

### 3 The Master Schedule Problem

In the previous section we considered the ideal situation that a senior nurse wants to create next week's schedule from scratch. However, a typical home health care service prefers the application of the same basic or master schedule from week to week. On the one hand, the advantages of this approach are twofold: first, it eases the senior nurse's work, and secondly, it ensures a high continuity in the schedules, i.e., the routes of the nurses do not change significantly, and the patients are treated by the same nurse. On the other hand, the continuous changes in the patient pool quickly lead to a situation in which the historic master schedule does not reflect the optimal routes any more. This may be costly for the service provider.

The master schedule problem (MSP) addresses the task to create optimal master schedules for the current patient pool. For those master schedules, the nurses are irrelevant, since they are assigned to the tours from week to week according to their availability. Therefore the task of the MSP is to generate a schedule with a minimal number of tours, the master tours, such that all jobs are performed. In the MSP, the jobs only have one possible shift combination.

For the tours we only have to consider two additional restrictions. First, a tour may only contain tasks of one qualification dimension, and secondly, all tours are restricted to a maximum length  $L$ . This tour length  $L$  reflects legal requirements in Germany. In Germany, a nurse has to take a break of 30 minutes if her tour exceeds a length of six hours. Consequently, the master tours are designed

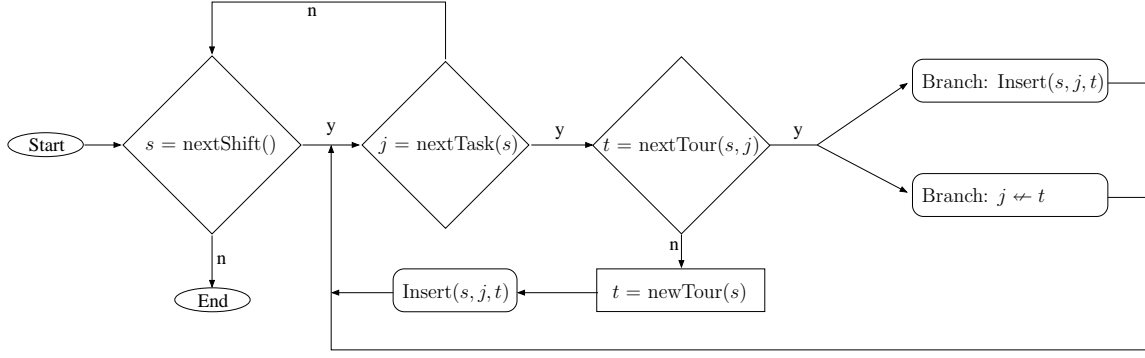


Figure 2: CP heuristic for the MSP

such that they are less than six hours.

The MSP is still *NP*-hard, although it is less complicated than the HHCP. Therefore we propose a constraint programming heuristic as solution approach analogously to the first stage in the HHCP.

In the course of this section, we present the deviations of the MSP model from the HHCP model. We conclude with the CP heuristic.

### 3.1 Model

For the MSP, the jobs and the depot are defined analogously to the HHCP with the restriction that each job only has one possible shift combination, i.e.,

$$\forall j \in \mathcal{J}: R_j = \{R_{j,1}\}.$$

Additionally, a set of possible qualifications  $\mathcal{Q} = \{q_1, \dots, q_Q\}$  is given. Each job  $j$  requires a qualification  $req_j$ . Now, we categorize the qualifications, i.e., either they are cleaning qualifications or nursing qualifications etc. Therefore we introduce a comparability among qualifications. We say that two qualifications  $q_r$  and  $q_s$  are comparable if they belong to the same kind of qualification. Otherwise, we call them parallel. A maximal set of comparable elements forms a qualification dimension.

Furthermore, a tour is a sequence of tasks in one shift that represents a feasible route. It has a maximal length of  $L$  and may only contain tasks of one qualification dimension.

The constraints for the MSP are also analogous to the HHCP with the addition of the maximal tour length and the restriction of the qualification dimension. Obviously all constraints regarding nurses are omitted.

### 3.2 Objective Function

The single goal is to minimize the number of necessary tours. Other possible objectives include the traveling distance and the required qualification to perform a tour, but those objectives pale in comparison to the number of tours, i.e., nurses, needed.

### 3.3 Solution Approach

Before we commence with the constraint programming heuristic, we sort all tasks in all shifts according to their earliest start time as in the initial solution heuristic for the HHCP. Then we proceed with the heuristic presented in Figure 2.

We start with zero tours in each shift. Now let  $s$  be the next shift with unscheduled tasks. If there is no more such shift, then the search is complete. Otherwise we consider the next unscheduled task  $j$  in  $s$ . After the task is selected, we determine a tour for it. This assignment decision is the key to the success of our heuristic. Therefore it is explained in more detail later on. After a tour  $t$  is

---

**Algorithm 2** getNextTour(Shift  $s$ , Task  $j$ )

---

```
1: minTourDistance =  $\infty$ 
2: minTourIndex = -1
3: for all  $t \in \mathcal{T}$  do
4:   if  $y_t \neq s$  then
5:     continue
6:   if  $tq_t \parallel req_j$  then
7:     continue
8:   if  $t \notin D(x_j^s)$  then
9:     continue
10:   $i$  = currently last task on tour  $t$ 
11:  minAdd =  $\max \left\{ d_{i,j} + pt_j, \right.$ 
            $\left. \min \{ D(st_j^s) \} + pt_j - (st_i^s + pt_i) \right\}$ 
12:  if  $dur_t + \text{minAdd} \leq L$  then
13:    if  $d_{i,j} < \text{minTourDistance}$  then
14:      minTourDistance =  $d_{i,j}$ 
15:      minTourIndex =  $t$ 
16:    else if  $d_{i,j} == \text{minTourDistance}$  then
17:      if  $dur_t > dur_{\text{minTourIndex}}$  then
18:        minTourDistance =  $d_{i,j}$ 
19:        minTourIndex =  $t$ 
20: return minTourIndex
```

---

selected for task  $j$ , we add the following two branches to the search tree: first, we try to append the current task to the tour (**Insert**( $s, j, t$ )); if this assignment fails, we set a marker that task  $j$  cannot be served in tour  $t$  ( $j \leftarrow t$ ). It may happen that none of the currently existing tours can be used to serve task  $j$ . Then we have to open a new tour for shift  $s$  and insert task  $j$  as its first task. This unavailability of a tour occurs for example, if no tour was created at all or no tour for the current task's qualification dimension exists. We proceed in this manner with the next task and the next shift, until all tasks in all shifts are assigned to a tour.

Finally, it remains to explain how the next tour is selected. This is shown in Algorithm 2. Its general idea is always to assign the task to the currently closest tour. We store the currently closest tour in the variable **minTourIndex** with the currently closest distance of **minTourDistance**. Keep in mind that the tasks are sorted by earliest start time. Therefore we only have to consider the case in which a task is appended to a tour.

To determine the best tour for task  $j$ , the algorithm traverses all tours  $t \in \mathcal{T}$ . A tour  $t$  is not allowed to perform  $j$  in the following three cases:

1. The tour is not open for the current shift (line 4), or
2. The current qualification required for the tour is incomparable to the qualification needed for the task, i. e.,  $tq_t$  is of a different qualification dimension than  $req_j$  (line 6), or
3. The tour is infeasible for the current task, since it is not in the task's domain (line 8). This case occurs for example if the tour was previously tried and this assignment led to an inconsistency.

If tour  $t$  passes all tests, we still have to determine whether the current tour suits task  $j$  better than all previous tours. Therefore we first calculate the minimal addition to the tour, if the current task  $j$  is appended (line 11). The following two cases need to be distinguished from one another (see Figure 3):

**Case 1:** Task  $j$  can start directly after the current last task, then we just have to add the traveling distance plus the processing time of the new task.

**Case 2:** Task  $j$  can only start after its time window has started, then we have to add the processing time of the new task plus the waiting time until it can start.

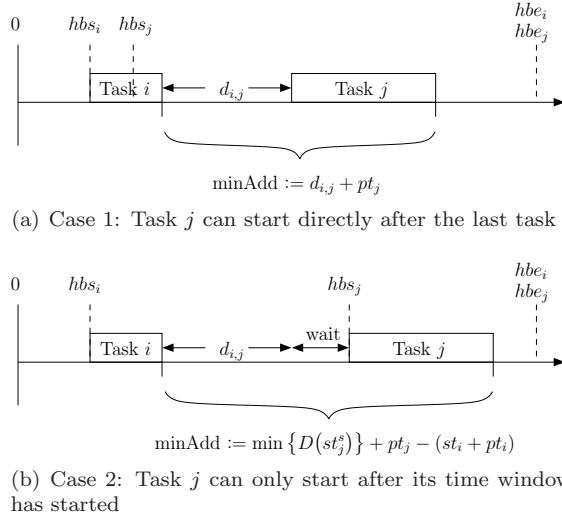


Figure 3: Determination of  $\text{minAdd}$

Now task  $j$  is only allowed in tour  $t$  if tour  $t$  plus task  $j$  still satisfies the maximal tour length constraint (line 12). If adding  $j$  is possible and the current tour is closer to the new task than any other tour (line 13), then we store the current tour as the currently best one (lines 14–15). For tie-breaking, we favor the tour with greater duration so far (line 17), keeping in mind that we prefer opened tours to be as long as possible for maximal utilization of the nurses.

If a `minTourIndex` of  $-1$  is returned, no feasible tour was found for task  $j$ , and hence a new tour has to be opened for it.

Notice however that Algorithm 2 does not necessarily return a feasible assignment of a task to a tour. The constraint programming layer checks the assignment for feasibility and may reverse it by backtracking.

The qualifications required to perform a job and the qualifications provided by a nurse are respected in this heuristic, but not implicitly incorporated. On the one hand, they are respected since only tasks of one qualification dimension share one tour. On the other hand, the heuristic does not minimize the necessary qualification for a tour.

## 4 The Operational Planning Problem

The task of the operational planning problem (OPP) is to incorporate last minute changes into an existing plan resulting from a solution to the home health care problem or an assignment of nurses to a master schedule. In practice, a senior nurse usually spends her Friday solving the OPP for the next week.

We call any event that requires a revision of the existing plan a perturbation. As perturbations, various events are imaginable, but generally they are divided into two groups: perturbations affecting the nurses and perturbations affecting the patients.

Examples for perturbations caused by nurses or affecting nurses are holidays, sickness of a nurse, or ad hoc team meetings. On the patients side, perturbations occur if a patient has no more demand, either temporarily or permanently, if a patient demands an alteration of his time window, if the service time for a patient changes, or if a new patient is supposed to be added to the schedule.

In the course of this section, we consider only the perturbation of a new patient, since it is the most crucial perturbation for the schedule and at the same time the most difficult one to

incorporate. The event is also of special interest to the home health care service, since it simplifies the negotiations with the patients concerning the time windows. After a patient has stated his preferred time window, the senior nurse evaluates the cost of granting the patient’s wish by solving an OPP consisting of the old schedule with the new patient as perturbation. This allows the service provider to decide whether it makes sense to accept the new patient under the current conditions or not.

All perturbations share the property that they are not online problems, i.e., they have to be considered tomorrow or the next week, but not immediately. In addition, they are too insignificant to justify the re-creation of the whole schedule by solving the HHCP, because a re-solved HHCP most likely leads to a very different plan with a high perturbation value.

The task of the OPP is to create an optimal schedule incorporating the perturbations. We measure the quality of a schedule by two criteria: on the one hand we seek to minimize the perturbation between the current solution and the initial one; on the other hand we still try to minimize the original objectives (number of unscheduled tasks, patient-nurse loyalty, overtime costs, and traveling distance). Consequently, the objective function of the OPP is a weighted sum of the perturbation penalty and the original objective function.

Several indicators exist to measure a perturbation. However, the inconveniences for the patients (as customers) are of greater importance than the inconveniences for the nurses, hence we choose a perturbation penalty that measures the perturbations from the patient’s point of view. For this purpose, two possibilities come to mind: a measurement of the alteration of the patient-nurse assignments or a measurement of the alteration of the start times for the patients. We believe that the patients prefer consistency in their service time to consistency in the assigned nurse, since patients are used to being visited by several nurses over the course of their treatment (due to nurse holidays or illnesses). Therefore we measure the perturbation of a new solution according to the change in starting times of patient visits.

Due to the complexity of the OPP, an exact method is incapable of computing a feasible solution as quickly as required. Thus, we develop a two stage heuristic approach to solve the OPP. First, we insert the new patient heuristically at its best position to quickly receive a feasible solution. Afterwards a tabu search heuristic improves the heuristic’s solution if additional computation time is available. We choose a tabu search heuristic since it has proven to be a successful approach for vehicle routing problems (see [3, 5]). Furthermore, it operates with small moves that do not disturb the current solution significantly, hence maintaining a low perturbation penalty.

In the course of this section, we introduce a function to measure the perturbation between two different solutions. With this function, we define the objective function, before we commence with the description of the two stage algorithm. For this purpose, we first describe the insertion heuristic and end with the tabu search algorithm.

## 4.1 Model

The operational planning problem is based on an instance of the home health care problem. Therefore the definitions for the nurses, jobs, etc. in the OPP are analogous to the ones in the HHCP. Furthermore a feasible, but not necessarily optimal, solution  $\hat{\sigma} = (\hat{r}, \hat{x}, \hat{st}, \hat{y}, \hat{job})$  is known for the initial problem.

Moreover we define the difference between any solution  $\sigma$  to the initial solution as the pertur-

bation penalty. The perturbation penalty  $\varphi_j^s$  for a task  $j$  in a shift  $s$  is given as:

$$\varphi_j^s = \begin{cases} 0 & , \text{ if } x_j^s = 0, \\ |st_j^s - \widehat{st}_j^s| & , \text{ if } x_j^s \neq 0, \widehat{x}_j^s \neq 0, \\ 0 & , \text{ if } x_j^s \neq 0, \widehat{x}_j^s = 0 \text{ and } \\ & st_j^s \in [hbs_j, hbe_j - pt_j] \\ \min \left\{ |st_j^s - hbs_j|, \right. & , \text{ if } x_j^s \neq 0, \widehat{x}_j^s = 0 \text{ and } \\ \left. |st_j^s + pt_j - hbe_j| \right\} & st_j^s \notin [hbs_j, hbe_j - pt_j]. \end{cases}$$

The definition of the perturbation penalty distinguishes between two main cases: the case in which job  $j$  is scheduled in shift  $s$  in solution  $\sigma$  ( $x_j^s \neq 0$ ) or not ( $x_j^s = 0$ ). If the job is unscheduled, a definition of a perturbation penalty is not reasonable, since an unscheduled job has an arbitrary start time. Hence the perturbation is set to 0 in this case.

If the job is scheduled in the current solution ( $x_j^s \neq 0$ ), we have to consider three cases:

1. Job  $j$  is also scheduled in the initial solution ( $\widehat{x}_j^s \neq 0$ ):

Then the perturbation penalty is the difference between the initial start time and the current start time.

2. Job  $j$  is unscheduled in the initial solution ( $\widehat{x}_j^s = 0$ ), but is now scheduled *within* its time window ( $st_j^s \in [hbs_j, hbe_j - pt_j]$ ):

Again, a definition of the perturbation value is not reasonable, since the job was previously unscheduled. Hence no penalty is charged.

3. Job  $j$  is unscheduled in the initial solution ( $\widehat{x}_j^s = 0$ ), but is now scheduled *outside* its time window ( $st_j^s \notin [hbs_j, hbe_j - pt_j]$ ):

This case is only possible for the new job, added as perturbation. For all other jobs, we enforce a hard time window, forbidding an assignment outside the time window. However, it is absolutely necessary to schedule the new patient. Therefore, we allow—in this case only—a soft time window. As a consequence, the new job can be scheduled at any time, but a penalty is raised if it is scheduled outside its time window. The penalty value indicates how far the job lies outside its time window.

## 4.2 Objective function

As mentioned before, the objective function for the operational planning problem consists of the original objective function evaluating the solution to the HHCP and a term representing the perturbation. Hence the objective function is

$$\min \quad \beta_1 \cdot \widehat{\zeta} + \beta_2 \cdot \sum_{s \in \mathcal{S}} \sum_{j \in \mathcal{J}} \varphi_j^s,$$

where  $\widehat{\zeta}$  represents the original objective function with the number of unscheduled tasks, patient-nurse loyalty, overtime and traveling distance, and the parameters  $\beta_1$  and  $\beta_2$  allow an emphasis on one or the other objective, i.e.,

$$\beta_i \in [0, 1] \quad \forall i = \{1, 2\} \text{ and } \beta_1 + \beta_2 = 1.$$

## 4.3 Solution Approach

For solving the operational planning problem, we choose a two stage approach: first, a heuristic inserts the new job at the currently best position. Then in the second stage, a tabu search meta-heuristic (see [7, 8]) improves the solution until a time limit or a move limit is reached.



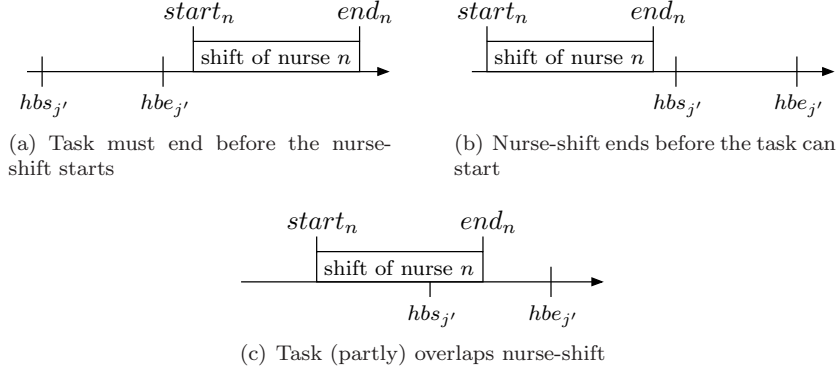


Figure 4: Three cases of the relation between a nurse-shift and the new task

As in the previous algorithms, a CP layer maintains feasibility of the assignments throughout the whole algorithm. Specifically, it indicates infeasible assignments as early as possible to prevent the exploration of infeasible branches of the search tree.

The insertion heuristic simply tries to insert the new job  $j'$  at all currently possible insertion points. For this purpose, it appends, prepends, or inserts the new job at the given tours. Afterwards the feasibility and the objective function value are evaluated by the constraint programming layer. However, the heuristic does not perform complex alterations of the schedule, e.g., the swapping of tasks or anything similar. Therefore only a few of the potential solutions are explored, but a quick determination is guaranteed.

The tabu search algorithm seeks to traverse the whole solution space by moving from a current solution to the best solution in a surrounding neighborhood. In contrast to a descent method, it allows deteriorated solutions to move away from local optima. A list of tabu moves prevents potential cycling. The neighborhood in each iteration is determined by a move. Therefore, a library of moves is available to diversify the search. Possible moves are discussed later on.

The two stage approach guarantees a balance between speed and solution quality. On the one hand, the insertion heuristic explores all current insertion points quickly due to constraint programming, and thus returns with a feasible solution within seconds. On the other hand, the tabu search spends time on advanced alterations of the routes to cope with the perturbation, and thus improves the quality of the existing solution. Therefore the senior nurse obtains a feasible solution quickly, but additionally has the possibility of receiving improved solutions if she has some computation time at her disposal. In this section, we first present the insertion heuristic before we discuss the tabu search and its moves.

For the insertion heuristic, the algorithm traverses all nurses that work in the corresponding shift  $s$ . If a nurse works shift  $s$ , three cases may occur: (a) the time window of the new task ends before the nurse-shift starts, (b) the nurse-shift ends before the time window of the task starts, or (c) the time window of the task (partly) overlaps the nurse-shift. The three cases are displayed in Figure 4.

In Figure 4(a), we simply prepend the new task to the existing tour. This is possible if

$$d_{0j'} \leq hbe_{j'} - pt_{j'}, \text{ and} \quad (5)$$

$$hbe_{j'} + d_{j', job_{n,1}^s} \leq start_n. \quad (6)$$

If inequality (5) is violated, then the new task cannot be scheduled as the first task in any tour due to an infeasible time window. In contrast, a violation of inequality (6) may still lead to a feasible solution if all tasks in the existing tour of nurse  $n$  can be scooted backwards. The CP layer evaluates the feasibility of this route alteration.

If the nurse-shift ends before the task time window starts (Figure 4(b)), we simply reverse tactics

and append the new task to the current tour. This alteration is possible if

$$end_n + d_{job_{n,k},j'} \leq hbe_{j'} - pt_{j'} \quad (7)$$

$$hbs_{j'} + pt_{j'} + d_{j',0} \leq H, \quad (8)$$

where  $k$  is the last task in the current route of nurse  $n$ . If inequality (7) is violated, a feasible route may still be found by an alteration of the start times of the tasks in the route, whereas a violation of inequality (8) indicates that no feasible solution exists at all for task  $j'$ .

The final case actually covers four cases of its own that all share the property that the current route and the task's time window overlap at least partly. The four cases are: the route encloses the time window completely, the time window encloses the route completely, the route starts before the time window but stretches into it (Figure 4(c)), and the route starts within the time window and stretches over the end of the time window.

In the overlapping case, the insertion is more difficult, since multiple insertions are imaginable (see Figure 5). The new task  $j'$  may be inserted in the route between the tasks  $\underline{k}$  and  $\bar{k}$  where task  $\underline{k}$

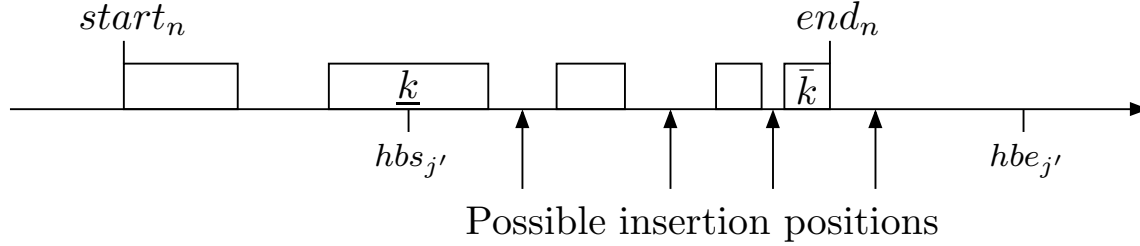


Figure 5: Detailed possibilities for the case of Figure 4(c)

is the first task to end after  $hbs_{j'}$  and task  $\bar{k}$  is the last task to start before  $hbe_{j'}$ . Additionally, the new task can be prepended or appended to the whole tour if inequality (6) or inequality (7) hold, respectively.

The complete insertion heuristic is omitted due to its length. We refer to [11] for a detailed description. In the heuristic, all discussed cases are evaluated to determine all possible insertion points for the new task. Thereafter, the insertion points are probed for their objective function value.

The tabu search algorithm requires as input the operational planning problem, an initial solution  $\sigma$  (provided by the insertion heuristic), and a list of possible moves  $\mathcal{M}$ . A move  $m \in \mathcal{M}$  determines the current neighborhood  $N_m(\sigma)$  of solution  $\sigma$ , where  $N_m(\sigma)$  is the set of possible new solutions reachable from  $\sigma$ . An example for a move is a swap of two tasks of two distinct routes. Then, the neighborhood corresponding to this move is the set of all solutions that can be reached from  $\sigma$  by swapping any two tasks of any distinct routes. A pseudo-code for the tabu search algorithm is displayed in Figure 3.

In the first line, we initialize the iteration counter  $k$ , the tabu list  $T$  with size  $\tau$ , and the currently best solution  $\sigma^*$ . Then the next iteration is started if the termination criterion is not satisfied. The termination criterion is usually a limit on the number of iterations or a time limit. Other possibilities include for example the criterion that the search is terminated if the best objective function value has not been improved for the last  $x$  iterations.

In the tabu search iteration (lines 3 to 10), the iteration counter is first incremented. Then we select a move  $m$  out of the possible moves  $\mathcal{M}$ . In a simple tabu search, only one move type is available, i.e.,  $|\mathcal{M}| = 1$ . Therefore the same type of neighborhood is considered in each iteration and the exploration of the search space is only guaranteed by the tabu list. An improved tabu search algorithm uses in contrast a set of different moves to diversify the search. In this case, the move for the current iteration is either chosen randomly or adaptive weights are determined to evaluate the success of the moves during the algorithm. Reasonable moves for the OPP are discussed later in this section.

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**Algorithm 3** Tabu Search Algorithm [7] for the OPP

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**Input:** OPP, initial solution  $\sigma$ , move list  $\mathcal{M} = \{1, \dots, M\}$

```
1:  $k = 0, T = \emptyset, \sigma^* = \sigma$ 
2: while termination criterion not satisfied do
3:    $k = k + 1$ 
4:   Select  $m \in \mathcal{M}$ 
5:   if  $N_m(\sigma) \setminus T \neq \emptyset$  then
6:      $\sigma_k = \operatorname{argmin}_{s \in N_m(\sigma)} \{\zeta(s)\}$ 
7:     if  $\zeta(\sigma_k) < \zeta(\sigma^*)$  then
8:        $\sigma^* = \sigma_k$ 
9:        $\sigma = \sigma_k$ 
10:    Update  $T$  with  $\sigma_k$ 
```

---

A new solution can be computed if the current neighborhood  $N_m(\sigma)$  includes solutions different from the tabu ones (line 5). If such solutions exist, we evaluate them to determine the optimal solution  $\sigma_k$  with respect to the objective function (line 6). The term  $\zeta(s)$  denotes the objective function value of solution  $s \in N_m(\sigma)$ .

If this locally optimal solution  $\sigma_k$  in  $N_m(\sigma)$  improves the overall best solution value (line 7), then we store  $\sigma_k$  as the currently best solution. In any case (independent of whether we found a new best solution or not) we remember  $\sigma_k$  as the current solution  $\sigma$  and update the tabu list  $T$ . The tabu list is implemented as a first in, first out (FIFO) list of size  $\tau$ , i.e., the last  $\tau$  moves are tabu. If  $T$  already has size  $\tau$  and a new solution  $\sigma_k$  has to be inserted, we remove the oldest solution from the tabu list and append the new one. Afterwards, the next iteration starts if the termination criteria are not yet met.

The key to a successful tabu search algorithm lies in the choice of moves and corresponding neighborhoods. For the operational planning problem, three different types of moves are reasonable: moves to improve the routes, moves for a general schedule improvement, and moves to reduce the perturbation. Since we are mainly interested in a low perturbation, we concentrate on moves out of this category.

Since we measure the perturbation exclusively by the offset of the starting times compared to the initial solution, we propose the following two tabu search moves:

1. Reassignment of a chain of shifted tasks:

The insertion of a new task may lead to a chain of early or late tasks if the whole route was scooted backwards so that the new task fits into it. If this chain can be assigned to a different nurse, the perturbation value decreases. Figure 6 shows an example of such a reassignment.

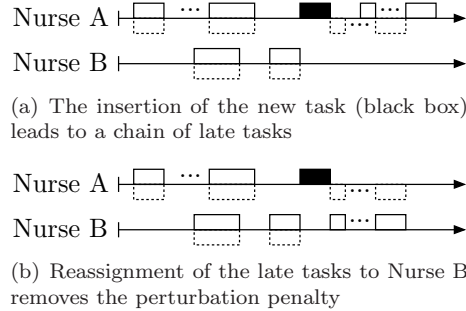


Figure 6: Example of a successful reassignment of a chain of late tasks  
(The dotted boxes underneath the time line represent the initial solution)

## 2. Removal of a single perturbed task:

In many cases, the schedule is too tight to reassign a whole chain of tasks as described above. Then, an improvement may be achieved if one task is removed from the route, allowing all tasks after it to move forward and be back on time. The removed task is inserted at its best new position or even temporarily marked as unscheduled. An example for this move is shown in Figure 7.

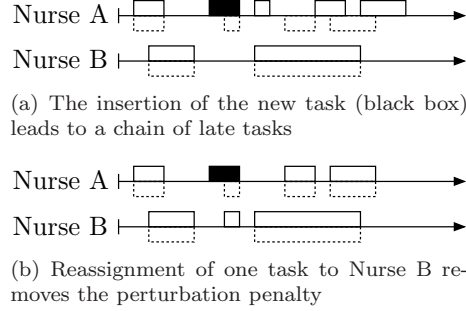


Figure 7: Example of a successful removal of a late task  
(The dotted boxes underneath the time line represent the initial solution)

## 5 Experiments with Real-World Data

We evaluated the capabilities of our solution approach with two real-world data sets. Instance 1 was provided by a German charity operating in a rural area, while instance 2 is from an urban area in the Netherlands. Instance 2 was kindly provided by the Dutch software company ORTEC. The size of the instances is compared in Table 3.

Instance	Days	Nurses	Jobs	Tasks
Data set 1	7	11	53	287
Data set 2	7	12	95	361

Table 3: Comparison of the size of the two real-world data sets

Both data sets provide the problem data for a week, i.e., the jobs that have to be performed and the nurses that are available. All parameters described in Section 2 are present for the jobs and the nurses. However, only one shift combination is given for each job.

At first sight, the test instances seem comparable according to their size, but the structure of the data sets is heterogenous. As stated before, data set 1 arises from a rural area of Germany, hence most traveling distances range from 15 to 45 minutes. In contrast, the urban data shows a maximal traveling distance of 15 minutes. A second difference lies in the length of the average service time. While most tasks in both instances only require short service times (between 15 and 60 minutes), instance 1 contains several jobs with a service time of more than three hours. This irregularity further complicates the solving of data set 1.

All computations were performed on an AMD ATHLON X2 processor with 2 GHz and 2 GB Ram. The screenshots of schedules are taken from ILOG SCHEDULER VIEWER 1.0 and the routes are displayed with GOOGLE MAPS.

In this section, we present computational results for the experiments with the real-world data sets. First, we compare for the HHCP the best solution of our hybrid approach to an actually worked plan for instance 1 and a proposed schedule by the ORTEC software for instance 2. Then we compute master schedules for both data sets with our CP heuristic. Unfortunately, we were not provided with any test instance for the OPP. Therefore no computational experiments are given for this problem.

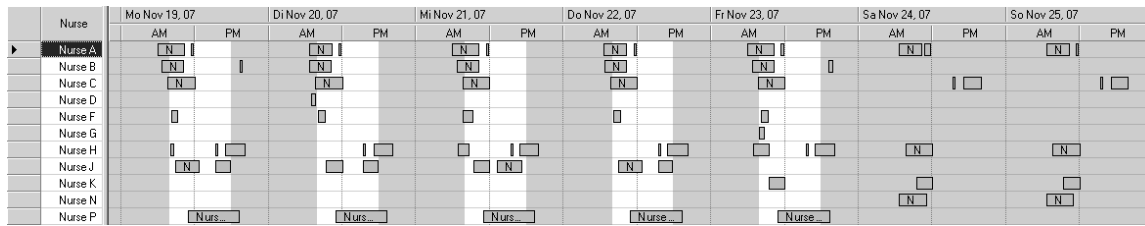
## 5.1 Experiments for the HHCP

We ran our hybrid algorithm for both real-world data sets with a move limit of 1000 moves and a time limit of two hours.

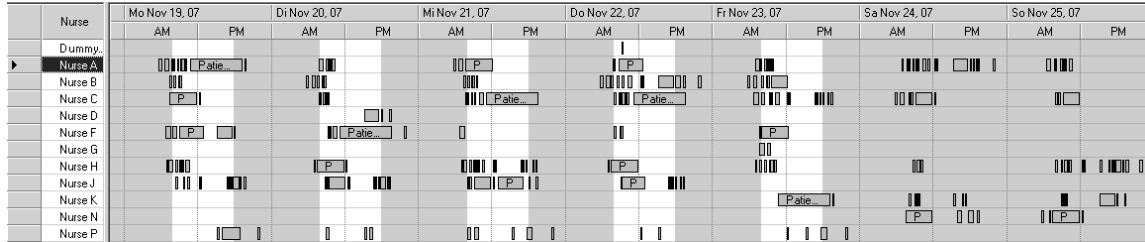
The first data set is based on an actual working plan of a home health care provider in a rural area of Germany. Therefore each job has a fixed time in which it is performed. To receive some degree of freedom for our algorithm, we created time windows for each job by allowing it to start 15 minutes earlier or later than in the actual schedule. Other than that, each job inherits its parameters from the actual plan, i.e., shift combination, its required qualification, and its service time.

Remember that the first data set is somewhat atypical for a home health care service, since it includes mostly jobs with a short service time (less than an hour), but also jobs with a long service time (three hours and more).

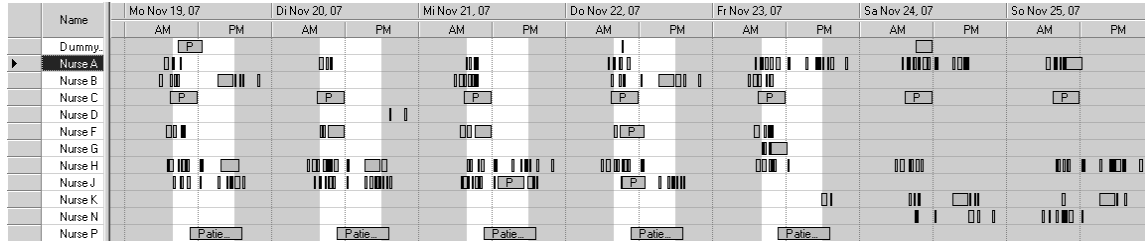
We expect this instance to be very hard to solve, because it does not contain the degrees of freedom in the instance that an algorithm needs to perform well. In our case, this freedom arises from the possibility to alter the shift combinations of the jobs or from the availability of generous time windows. Instead, we ask the algorithm to find the one solution that the service provider applied.



(a) Actual schedule worked by the home health care service



(b) Schedule created by the ALNS



(c) Schedule created by the ALNS after two long jobs were fixed in advance

Figure 8: Comparison of solutions for Monday of data set 1 generated by the ALNS compared to the actually worked plan

In Figure 8, we present three schedules for the given instance. The first schedule (Figure 8(a)) represents the actually worked plan by the service provider. The columns show each day of the week from Monday through Friday, while the rows stand for the available nurses. The blocks represent the time when the nurses worked. For example, Nurse A worked Monday morning from 6:30 a.m. to 10:30 a.m. Then she had a little break and worked again from 11:30 a.m. to 12:00 p.m. We do

not have information about the actual tasks accomplished during this time. Therefore we cannot calculate any term of the objective function for this schedule, despite the total working time of the nurses.

The ALNS was terminated after two hours and 550 moves. Its best schedule is displayed in Figure 8(b). In the schedule, the blocks represent actual tasks at patients. The wider the block, the longer the duration of the task. Due to the zoom level, most jobs only appear as bars. The ALNS was able to find a plan for the instance that leaves only four ten minute tasks unscheduled. All of these unscheduled tasks occur on Thursday morning. Because they partly overlap in time, they appear on Thursday morning as one bar in the first row at the “dummy nurse”. Otherwise all tasks were scheduled with the cost of 8745 overtime units.

The computed schedule is a reasonable plan for this data set. The assignment of tasks to nurses is such that the shifts are mostly very tight, i.e., no long breaks occur. Only the shifts of Nurse J show longer breaks. However, it seems that the overtime is not equally split: for example Nurse A has a very long shift on Mondays. Additionally, it is obvious that the tasks with the long service times are not always assigned to the same nurse, which seems to be advisable for a good plan.

The obtained results and the occurring overtime costs can be explained by the following considerations. First of all, the ALNS did not receive any information about the specific originalities of this instance. As mentioned before, the occurrence of eight long jobs complicates the search for an optimal solution. In practice, these jobs are assigned to a special nurse that sometimes only works this job. An example for this can be seen in Figure 8(a), where the shifts of Nurse P only consist of one patient. By fixing only two of these long jobs in advance, we can significantly simplify the instance and receive a better plan. The plan obtained after these modifications to the instance is shown in Figure 8(c).

In the modified plan, the same number of tasks remain unscheduled, but the overtime costs are reduced to 8180. Moreover, the plan has a much better structure. For example the unfavorable shifts of Nurse P are replaced by the a priori assigned long task. This leads to an improved placement of the nurse’s previous tasks in existing routes of other nurses. The schedule for the modified instance partly shows the same structure as the actual plan. For example Nurse J has the same split services assigned. We expect the plan to become better with any additional fixation of long jobs.

Another reason why we could not recreate the actual plan was the use of split services by the service provider. A split service of a nurse occurs if she starts her shift and then interrupts it for some time. Split services can be seen in the actual plan for example in every shift of Nurse A or for Nurse H. Our model does not allow split services. Therefore the break is taken into account as working time which partly explains the huge amount of overtime. If we look at the Tuesday shift of Nurse J in the actual plan, we spot a break of three hours and 15 minutes. This break is misleadingly accounted for an overtime of 195 time units in the ALNS schedule.

Other reasons for the overtime stem from differing assumptions of traveling distances between tasks. While we derive our traveling distances by rounding the time distance from GOOGLE MAPS to the next “5” (i.e., a traveling distance of 13 becomes 15), the service provider manually sets the traveling distance based on experience. In average, these traveling distances are lower, and thus allow a tighter schedule.

Finally it seems possible that some last minute cancellations of tasks occurred in the actual plan, hence reducing the required nurse working times. Especially on Saturday and Sunday afternoon, the actual plan has less tasks scheduled than the instance to our solution.

An example for a geographical representation of the created routes is provided with Figure 9.

The second data set is also based on actual data from a home health care provider. This time, the provider is located in an urban area of the Netherlands. To compare the results of our algorithm, we received a schedule created for the data by ORTEC’s software for home health care planning.

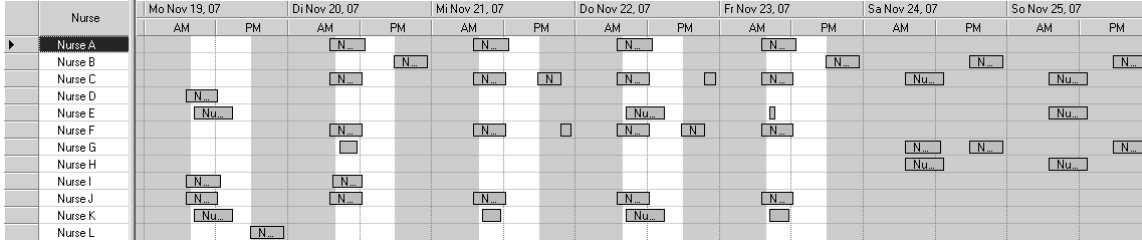
To fit instance 2 to our model, we split each day into a morning and an evening shift, since in the provided data the nurses’ availabilities are restricted to time windows.

Compared to the first data set, the second data set is expected to be easier solvable due to two reasons. First, the data is more homogenous, i.e., all tasks last between 15 and 60 minutes. Furthermore, each job comes with a time window that is significantly wider than the time windows in the first instance.

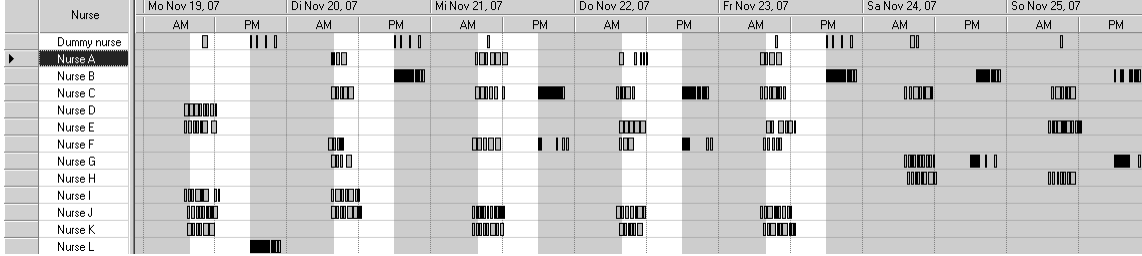




Figure 9: Example of three routes created for Sunday morning by the ALNS



(a) Schedule created by the professional software



(b) Schedule created by the hybrid algorithm

Figure 10: Comparison of the professional software’s schedule and the hybrid algorithm’s suggestion for data set 2

Figure 10 compares the results of our hybrid algorithm with the schedule created by the professional software. For the schedule in Figure 10(a), we do not possess information about which tasks are actually scheduled for a certain nurse. Therefore, the blocks in the schedule represent the working time of a nurse. For example, Nurse D works on Monday from 7:00 a.m. to 12:30 p.m., but we do not know which tasks. Consequently, a comparison of the patient-nurse loyalty and the traveling distance is impossible. However, it is possible to compare the number of unscheduled tasks and the overtime. The professional software was unable to schedule 55 out of 361 tasks (the unscheduled tasks are not shown in the schedule). Since in this plan all nurses are scheduled according to their hard time windows, no overtime occurs.

In our plan (Figure 10(b)), the best solution of the hybrid algorithm after two hours is displayed. All tasks are shown separately (represented by the bars). The split between morning and evening shift can be clearly seen. In the first row, labeled “Dummy nurse”, all unscheduled tasks are shown. Our algorithm fails to schedule only 19 out of 361 tasks. This solution contains no overtime.

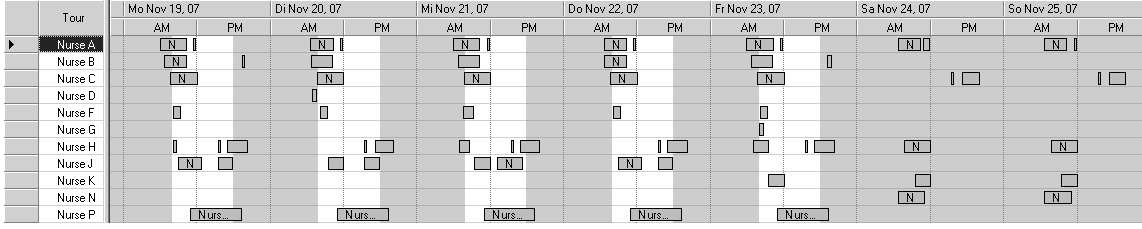
Compared to ORTEC’s plan, we were able to schedule 36 additional tasks. However, recall that the results are not directly comparable due to our modifications to the instance. It seems that the improvement was possible due to the modified nurse availability: in our instance, it is possible that a nurse works more in one shift than in the provided plan. The non-existent overtime indicates though that she does not work more throughout the whole schedule, hence she has to work less in another shift.

## 5.2 Experiments for the MSP

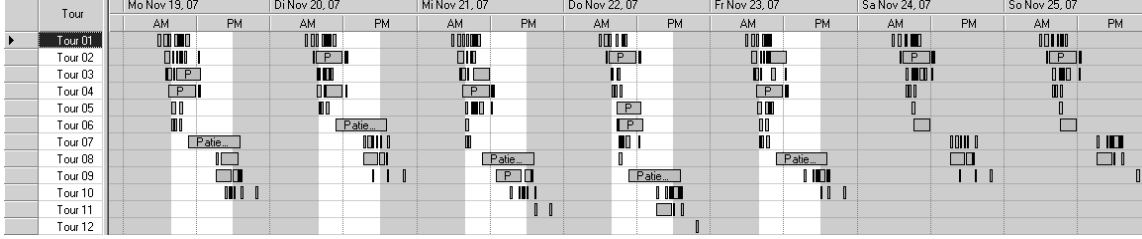
We tested our constraint programming heuristic for the CP with the same data sets as used for the HHCP. For all tests, a maximal tour length of six hours is assumed, i.e.,  $L = 360$ . The CP heuristic computed the solution for instance 1 in 6.7 seconds, while it took 19.5 seconds to compute the solution for the second instance.

Before we evaluate the results of our heuristic with the test data, it is important to notice that our heuristic’s results are difficult to compare with the provided schedules. In the actually worked plan for the first data set (see Figure 11(a)), the highly specialized assignment of nurses to patients conflicts with the general rule to obtain master tours with a maximal (and optimal) tour length.





(a) Actual schedule worked by the home health care service



(b) Master schedule created by the CP heuristic

Figure 11: Comparison of the actual plan to the suggested MSP solution for data set 1

This specialization leads to untypical assignments of nurses to only one short task (for example all assignments for Nurse F) or to split services (like all assignments for Nurse J).

For the second data set, the differences are even more significant and prevent a reasonable comparison of the results. The main difference is the small time window of availability for all nurses, hence all nurses work significantly less than six hours. Furthermore, a large amount of tasks remains unscheduled in the provided solution, while all tasks are assigned in any solution to the MSP.

For data set 1, a comparison of the actually worked schedule to the suggested MSP solution by the CP heuristic is shown in Figure 11. As in the previous section, the actually worked schedule only presents the working times of the nurses, while in the heuristic's schedule all performed tasks are displayed individually.

The CP heuristic computed a reasonable plan for the instance that could be used as a master plan. All tours are tight, and breaks occur rarely. Additionally, the maximal tour length was mostly utilized well. For example, on Monday, the first four tours all have a length of almost six hours. If short tours were created, they result from overlapping time windows: for example, Tour 05 and 06 on Monday contain only two or three tasks, respectively. On the one hand, some negotiations with the patients and resulting modifications of their time windows properly lead to the saving of a tour. On the other hand, those tours with about half of the maximal tour length are optimal tours for part-time nurses.

However, the heuristic's plan leaves some room for improvement. For example, the tours that only consist of one task (like Tour 12 on Thursday) should be avoided. It seems though that this may stem from the traveling distances we use. Most of these single task tours are easily added to another tour, only exceeding the six hour tour limit slightly.

Table 4 compares the number of tours actually used by the home health care service provider (HHCSP) to the number of tours required by the CP heuristic's (CPH) solution.

In the first row, the table shows the number of tours that the HHCSP used for the shifts Monday through Sunday. In total, 47 tours were run. The solution of the CP heuristic is displayed in the last row. Our algorithm required 70 tours in total.

At first sight, this seems an enormous discrepancy, but a closer look at the tours reveals the reasons. The HHCSP heavily utilized split services for their tours. Our algorithm does not allow split services, since we were told they are highly undesired by the nurses. Therefore it is unfair to compare the number of tours in the HHCSP plan to the number of tours in the heuristic's solution.

	Mo	Tu	We	Th	Fr	Sa	Su	Total
HHCSP	7	8	7	7	8	5	5	47
HHCSP*	10	9	9	8	10	6	6	58
CPH	10	9	11	12	10	9	9	70

Table 4: Comparison of the number of tours required for the actually worked plan and the master schedule proposed by the CP heuristic

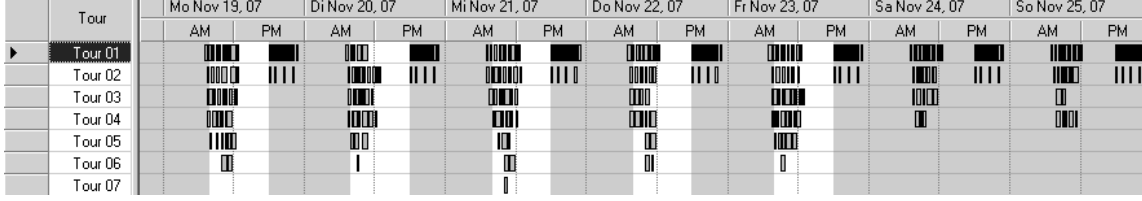


Figure 12: Master schedule created by the CP heuristic for data set 2

Thus we added a row HHCSPP\* in Table 4 that displays the results if a strict tour length limit of six hours is enforced for the HHCSPP solution.

For HHCSPP\*, we split each tour that was longer than 360 minutes in two tours. Then, the plan consisted of the displayed numbers of tours and a total of 58 tours. This is already much closer to the amount of tours that our CP heuristic calculated.

The remaining difference between the computer generated plan and the actually worked plan is likely based on the already mentioned discrepancies in the traveling distances. Furthermore, it is possible that due to last minute cancellations some tasks in our instance were not actually performed by the service provider. This seems to be especially true for the weekend, where the CP heuristic had to deal with significantly more tasks than are present in the service provider’s schedule.

Due to the incomparability of our heuristic’s solution to the provided solution, we only present the master schedule generated by our CP heuristic in Figure 12. The computed master schedule is an appropriate schedule for the instance, since it proposes tours that are tightly packed with tasks. In total, it calls for 53 tours to visit all patients. Hence it is obvious that in the corresponding HHCP, we have to end up with unscheduled tasks, since only 45 nurse-shifts are available. Therefore many of the unscheduled tasks in the ALNS solution for the HHCP (displayed in the row “Dummy nurse” in Figure 10(b)) are scheduled after the addition of a single nurse. This is true for example for the evening shifts of Monday, Tuesday and Friday, and also for Saturday morning.

A further reduction of the number of tours in the master schedule seems achievable by re-negotiating some of the patients’ time windows. If we look at the morning shifts of Tuesday, Wednesday and Friday, we see that single tasks require a tour on their own. In those cases, a re-negotiation of a single patient’s time window promises a reduction of the number of necessary tours, and therefore a reduction of the number of necessary nurses.

## 6 Conclusion

In this paper we have considered different aspects of planning problems for home health care services. We looked at the so-called home health care problem which seeks for a weekly (or more general a periodically) optimal plan. Since the current practice relies on a historic data based manual planning, we also looked into the master schedule problem which has to be updated in case of changes by using the operational planning problem.

In order to be able to provide quickly a feasible solution and improve this solution depending on the time available, we chose a two stage approach for two of the three problems. First, we construct a feasible solution (using constraint programming techniques) and in a second step we apply different improvement (meta-) heuristics. For the MSP we construct a feasible solution in one

step. Here, one could think about a feedback loop from the OPP to improve the master schedule if needed.

By using real-world data we could show the benefit of using more sophisticated planning methods in the home health care context.

In a next step we are planning to integrate all this methods into a decision support system for home health care planning. This will require a good integration with GI-Systems as well as a user friendly interface for decision making. Such a system could then be used to quantify the economic consequences of certain decisions. As demand is currently higher than the capacities of the home health care providing organizations the question which customer to include next from the waiting list has to be answered adequately. A decision support system could help to take the medical, the logistical and also the economical perspective into account.

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